## 6.2 Partial Fractions

EVALUATE THE FOLLOWING BY PARTIAL FRACTIONS.

**1264.** $\displaystyle\int \frac{1}{x^2-9}\,dx$

**1269.** $\displaystyle\int_{1/2}^{1} \frac{x+4}{x^2+x}\,dx$

Solution to 1264 :

Let $I = \displaystyle\int \frac{1}{x^2-9}\,dx$

The fraction $\dfrac{1}{x^2-9}$ can be written as

$$\frac{1}{x^2-9} = \underbrace{\frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}}_{\text{multiply both sides by } (x-3)(x+3)}$$

$$1 = A(x+3) + B(x-3)$$

To find A, substitute in the eq. $x = 3$

$$1 = A(3+3) = 6A \Rightarrow A = \frac{1}{6}$$

To find B, substitute in the eq. $x = -3$

$$1 = B(-3-3) = -6B \Rightarrow B = \frac{-1}{6}$$

Therefore,

$$\frac{1}{x^2-9} = \frac{1}{6}\left(\frac{1}{x-3}\right) - \frac{1}{6}\left(\frac{1}{x+3}\right)$$

$$I = \int \frac{1}{x^2-9}\,dx = \int \left[\frac{1}{6}\left(\frac{1}{x-3}\right) - \frac{1}{6}\left(\frac{1}{x+3}\right)\right] dx$$

$$= \frac{1}{6}\int \frac{dx}{x-3} - \frac{1}{6}\int \frac{dx}{x+3}$$

Remember

$$\int \frac{dx}{x+a} = \ln|x+a| + C$$

$$= \frac{1}{6}\ln|x-3| - \frac{1}{6}\ln|x+3| + C$$

$$= \frac{1}{6}\left[\ln|x-3| - \ln|x+3|\right] + C$$

$$= \frac{1}{6}\ln\left|\frac{x-3}{x+3}\right| + C \quad \#$$